Chapter 13: Reinforcement Learning

CS 536: Machine Learning

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Administration

Midterms due

Daily Show

Video

Reinforcement Learning

[Read Chapter 13]

- [Exercises 13.1, 13.2, 13.4]
- Control learning
- Control policies that choose optimal actions
- Q learning
- Convergence

Control Learning

Consider learning to choose actions, like:

- Robot learning to dock on battery charger
- Learning to choose actions to optimize factory output
- Learning to play Backgammon

Problem Characteristics

Note several problem characteristics:

- Delayed reward
- Opportunity for active exploration
- Possibility that state only partially observable
- Possible need to learn multiple tasks with same sensors/effectors

One Example: TD-Gammon

[Tesauro, 1995]

Learn to play Backgammon

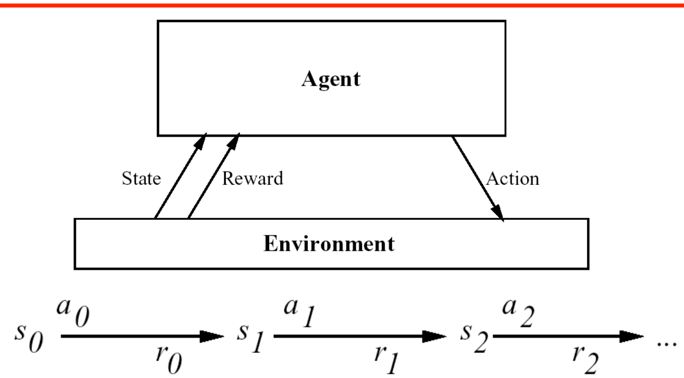
Immediate reward

- +100 if win
- −100 if lose
- 0 for all other states

Trained by playing 1.5 million games against itself.

Now, approximately equal to best human player.

The RL Problem



Goal: Learn to choose actions that maximize

$$r_0 + \gamma\, r_1 + \gamma^2 r_2 + \dots, \text{ where } 0 \le \gamma < 1$$

Markov Decision Processes

Assume

- finite set of states S; set of actions A
- at each discrete time agent observes state s_t in S and chooses action a_t in A
- then receives immediate reward r_t & state changes to s_{t+1}
- Markov assumption:
  - $r_t = r(s_t, a_t)$ and $s_{t+1} = \delta(s_t, a_t)$ depend only on *current* state and action
  - δ and r may be nondeterministic
  - δ and r not necessarily known to agent

Agent's Learning Task

Execute actions in environment, observe results, and

- learn action policy $\pi: S \rightarrow A$ that maximizes

$$E[r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \ldots]$$

  from any starting state in S
- here $0 \leq \gamma < 1$ is the discount factor for future rewards

Different Learning Problem

Note something new:

- Target function is $\pi: S \rightarrow A$
- but we have no training examples of form $<s, a>$
- training examples are of form $<<s, a>, r>$

Value Function

To begin, consider deterministic worlds...

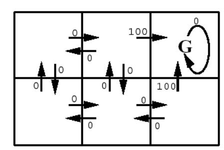
For each possible policy π the agent might adopt, we can define an evaluation function over states

$$V^{\pi}(s) \equiv r_t + \gamma\, r_{t+1} + \gamma^2\, r_{t+2} + \ldots$$
$$\equiv \Sigma_{i=0}^{\infty} \gamma^i\, r_{t+i}$$

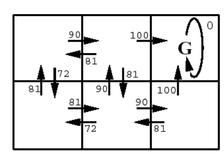
where $r_t, r_{t+1}, \ldots$ are generated by following policy π starting at state s.

Restated, the task is to learn the optimal policy π^*: $\pi^* \equiv \text{argmax}_{\pi}\, V^{\pi}(s), (\forall s)$.

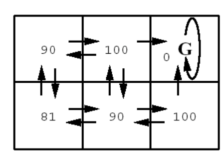
Example MDP



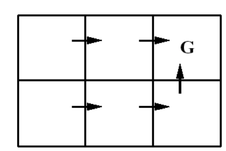
$r(s, a)$ (immediate reward) values



$Q(s, a)$ values



$V^*(s)$ values



One optimal policy

What to Learn

We might try to have agent learn the evaluation function $V^{\pi*}$ (we write as V^*)

It could then do a lookahead search to choose best action from any state s because

$$\pi^*(s) = \text{argmax}_a \, [r(s, a) + \gamma V^*(\delta(s, a))]$$

A problem:

- This works well if agent knows $\delta: S \times A \rightarrow S$, and $r: S \times A \rightarrow \Re$
- But, when it doesn't, it can't choose actions this way.

Q Function

Define new function very similar to V^*

$$Q(s, a) \equiv r(s, a) + \gamma V^*(\delta(s, a))$$

If agent learns Q, it can choose optimal action even without knowing δ!

$$\pi^*(s) = \text{argmax}_a \, [r(s, a) + \gamma V^*(\delta(s, a))]$$

$$= \text{argmax}_a \, Q(s, a)$$

Q is the evaluation function the agent will learn.

Training Rule to Learn Q

Note Q and V^* closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

This allows us to write Q recursively as

$$Q(s_t, a_t) = r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t))$$

$$= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a')$$

Nice! Let $\hat{Q}$ denote learner's current approximation to Q. Use training rule

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

where s' is the state resulting from applying action a in state s.

Q Learning in Deterministic Case

For each s, a initialize table entry $\hat{Q}(s, a) \leftarrow 0$

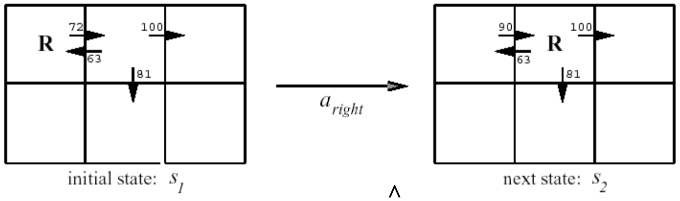
Observe current state s.

Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s, a)$ via:

  $$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$
- $s \leftarrow s'$

Updating $\hat{Q}$



$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$

$\leftarrow 0 + 0.9 \max\{63, 81, 100\} = 90$

notice if rewards non-negative, then

$$(\forall s, a, n)\ \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

and

$$(\forall s, a, n)\ 0 \leq \hat{Q}_n(s, a) \leq Q(s, a)$$

Convergence Proof

$\hat{Q}$ converges to Q. Consider case of deterministic world where see each $<s, a>$ visited infinitely often.

Proof: Define a full interval to be an interval during which each $<s, a>$ is visited. During each full interval the largest error in $\hat{Q}$ table is reduced by factor of γ

Let $\hat{Q}_n$ be table after n updates, and n be the maximum error in $\hat{Q}_n$; that is

$$\Delta_n = \max_{s,a} |\hat{Q}_n(s, a) - Q(s, a)|$$

Proof Continued

For table entry $\hat{Q}_n(s, a)$ updated on iteration $n+1$, the error in the revised estimate $\hat{Q}_{n+1}(s, a)$ is

$|\hat{Q}_{n+1}(s, a) - Q(s, a)|$

$= |(r + \gamma \max_{a'} \hat{Q}_n(s', a')) - (r + \gamma \max_{a'} Q(s', a'))|$

$= \gamma\ |\max_{a'} \hat{Q}_n(s', a') - \max_{a'} Q(s', a')|$

$\leq \gamma \max_{a'} |\hat{Q}_n(s', a') - Q(s', a')|$

$\leq \gamma \max_{a', s''} |\hat{Q}_n(s'', a') - Q(s'', a')|$

$|\hat{Q}_{n+1}(s, a) - Q(s, a)| \leq \gamma\, \Delta_n$

Note that we used the fact that

$|\max_a f_1(a) - \max_a f_2(a)| \leq \max_a |f_1(a) - f_2(a)|$

Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine V, Q by taking expected values

$$V^{\pi}(s) \equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

$$\equiv E[\Sigma_{i=0}^{\infty} \gamma^i r_{t+i}]$$

$$Q(s, a) \equiv E[r(s, a) + \gamma V^*(\delta(s, a))]$$

Nondeterministic Case

Q learning generalizes to nondeterministic worlds

Alter training rule to

$$\hat{Q}_n(s, a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n [r + \gamma \max_{a'} \hat{Q}_{n-1}(s', a')]$$

where

$$\alpha_n = 1/(1 + visits_n(s, a)).$$

Can still prove convergence of $\hat{Q}$ to Q [Watkins and Dayan, 1992].

Temporal Difference Learning

Q learning: reduce discrepancy between successive Q estimates

One step time difference:

$$Q^{(1)}(s_t,a_t) \equiv r_t + \gamma \max_a Q(s_{t+1},a)$$

Why not 2 steps?

$$Q^{(2)}(s_t,a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a Q(s_{t+2},a)$$

Or n?

$$Q^{(n)}(s_t,a_t) \equiv r_t + \ldots + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a Q(s_{t+n},a)$$

Blend all of these:

$$Q^{\lambda}(s_t,a_t) \equiv (1-\lambda) [Q^{(1)}(s_t,a_t) + \lambda Q^{(2)}(s_t,a_t) + \ldots]$$

Temporal Difference Learning

$$Q^{\lambda}(s_t,a_t) \equiv (1-\lambda) [Q^{(1)}(s_t,a_t) + \lambda Q^{(2)}(s_t,a_t) + \ldots]$$

Equivalent expression:

$$Q^{\lambda}(s_t,a_t) \equiv r_t + \gamma [(1-\lambda) \max_a \hat{Q}(s_t,a_t) + \lambda Q^{\lambda}(s_{t+1},a_{t+1})]$$

TD(λ) algorithm uses above training rule

- Sometimes converges faster than Q learning (not well understood in control case)
- converges for learning V for any $0 \leq \lambda \leq 1$ (Dayan, 1992)
- Tesauro's TD-Gammon uses this algorithm to estimate the value function via self play.

Subtleties & Ongoing Research

- Replace $\hat{Q}$ table with neural net or other generalizer
- Handle case where state only partially observable
- Design optimal exploration strategies
- Extend to continuous action, state
- Learn and use $\hat{\delta}: S \times A \rightarrow S$
- Relationship to dynamic programming and heuristic search